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Extended Command Governors for Constraint Enforcement in Dual-Stage Processing Machines

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Abstract— **This manuscript presents a scheme for the constrained control of a dual-stage system used in precision manufacturing. The system consists of two stages, a fast and a slow stage, whose actuators have different bandwidths. The fast stage is primarily constrained in its range of operation, and the slow stage is primarily constrained in allowable velocity and acceleration. The constrained control is based on the extended command governor, which is a constraint-enforcement scheme used for closed-loop systems subject to state and control constraints. A method of dividing the motion between fast and slow stages is presented which is based on tracking a minimal-motion reference for the slow stage. The extended command governor scheme is modified to simultaneously ensure constraint-admissible tracking of the minimal-motion reference and machining of the desired manufacturing pattern. Numerical simulation results are reported, showing successful tracking and constraint enforcement.**

I. Introduction

Multi-stage machines are composed of multiple parts, or stages, with different operating range and actuator bandwidths. In this work, we consider a precision manufacturing problem of controlling a dual-stage processing machine that consists of two dual-axis components. The components of the machine are coordinated with the aim of precisely machining a desired pattern onto a workpiece. One component moves a worktool into place in order to machine the pattern onto the workpiece; the other component moves the workpiece. To achieve good precision, the first component has a large bandwidth and produces fast accelerations but has limited operating range; it is referred to as the fast stage. The second component, that is the slow stage, moves more slowly but has a large operating range.

Specifically, the slow stage moves a workpiece, and the fast stage machines a pattern consisting of target points. The fast stage is dynamically decoupled from the slow stage and is designed to move quickly and precisely, resulting in a smaller range of motion. The operating range of the system is effectively increased by the large range of motion of the slow stage, but due to the size and weight of the slow stage, its acceleration and velocity are limited.

These mechanical limitations impose dynamic constraints on the system. The limited operating range of the fast stage imposes constraints on its position. The limits of motion of the slow stage mainly impose constraints on its velocity and acceleration. To enforce these constraints, we develop an extended command governor scheme that achieves the dual purpose of enforcing system constraints, while guaranteeing precise machining of the desired pattern. Control of dual-stage systems has been considered previously, *e.g.*, in the control of disk drives [1], [2], but without explicitly taking constraints into account.

Extended command governors (ECGs) [3] are schemes that modify a desired reference input to a linear closed-loop system, minimally altering the reference according to an objective function in order to enforce system output constraints. The conventional ECG was introduced as an extension to the command governor [4], [5] and the reference governor [6], [7], which are schemes that are used to enforce constraints in systems that track a reference [8]. Ordinarily, the ECG and related schemes are designed with the assumption that the desired reference signal is not known in the future; hence it is assumed that the desired reference stays constant over the prediction horizon. In this work, we slightly modify the conventional ECG for tracking a reference that varies along the prediction horizon. In the case of the dual-stage machine, the constraints that must be enforced by the ECG correspond to the slow-stage position, velocity and acceleration limits, and constraints on the range of the fast stage.

The desired pattern is generated so that it can be quickly and efficiently machined by the fast stage, and limitations on the motion of the slow stage are not considered when generating it. Since the operating range of the fast stage is limited and the machined pattern is the sum of the slow and fast stage positions, the slow stage must move in coordination with the fast stage. Previously, frequency separation has been used to split the desired reference between the slow and fast stages [9], but the presence of tight constraints that are handled only *a posteriori* makes this method suboptimal. In practice, all that is required is that the slow stage move the workpiece into the operating range of the fast stage. In this work, we present a method of computing a minimal-motion, slow-stage reference that satisfies the operating range constraint. The result is that the slow-stage moves minimally, while ensuring that the fast stage remains within operating range of the desired pattern.

This work is similar to [10]–[12], which considered the application of model predictive control (MPC), coupled with a reference governor scheme, to the control of dual-stage precision manufacturing systems. MPC is similar to the ECG in that it selects a sequence of future control inputs by solving a constrained optimization problem. In the previous work, MPC was used to improve a constraint-feasible reference sequence that was computed using the reference governor. The main difference between the former approaches and

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what is proposed in this work, is that the ECG solves the feasible reference generation and the trajectory optimization simultaneously in a single step, without the use of MPC or the reference governor. This is one of two of the main contributions of this paper; the other is the development of a minimal-motion constraint-feasible path for the slow-stage desired reference.

The paper is structured as follows. In Section II, we describe the ECG. In Section III, we describe the control problem. In Section IV, we describe our control scheme. In Section V, we present numerical results. The final section is the conclusion.

II. EXTENDED COMMAND GOVERNORS

The extended command governor (ECG) is an add-on predictive control scheme that ensures reference tracking for discrete-time systems in the presence of input and state constraints. The ECG is applied to linear, discrete-time systems of the form,

$$x(t+1) = Ax(t) + Bv(t), \tag{1a}$$

$$y(t) = Cx(t) + Dv(t) \in Y, \tag{1b}$$

where $x(t)$ is the n-dimensional state, $v(t)$ is the m-dimensional reference input, $y(t)$ is the p-dimensional output, and Y is polyhedral.

Given a desired reference $r(t)$, the ECG solves a quadratic programming (QP) problem in order to obtain a constraint-admissible reference $v(t)$ that is close to $r(t)$. The problem is designed so that $v(t)$ is the output of the following auxiliary system,

$$\bar{x}(t+1) = \bar{A}\bar{x}(t), \tag{2a}$$

$$v(t) = \bar{C}\bar{x}(t) + \rho(t), \tag{2b}$$

where $\bar{x}(t)$ is an $\bar{n}$-dimensional auxiliary state and $\rho(t)$ is an m-dimensional auxiliary input. The matrix $\bar{A}$ satisfies the Lyapunov condition $\bar{A}^{\mathrm{T}}P\bar{A} - P = -Q$ for some positive definite matrices P and Q. Combining (1) and (2), we obtain the closed-loop system,

$$\tilde{x}(t+1) = \tilde{A}\tilde{x}(t) + \tilde{B}\rho(t), \tag{3a}$$

$$y(t) = \tilde{C}(t)\tilde{x}(t) + D\rho(t) \in Y, \tag{3b}$$

where $\tilde{x}(t) = (x(t), \bar{x}(t))$, and the matrices $\tilde{A}$, $\tilde{B}$, and $\tilde{C}$ are appropriately defined. Using (3), we can construct a set of all constraint-admissible initial-state/constant-reference pairs using the method from [13]. This set is defined as,

$$\tilde{O}_\infty = \{(x_0, \bar{x}_0, \rho) : \tilde{x}(0) = (x_0, \bar{x}_0), \rho(t) = \rho, \text{ (3) is satisfied for all } t \in \mathbb{Z}_+\}. \tag{4}$$

For a constant reference input $\rho(t) \equiv \rho$, the steady-state output $y(t)$ is equal to $y_{ss}(\rho) := (\tilde{C}(I_{n+\bar{n}} - \tilde{A})^{-1}\tilde{B} + D)\rho = (C(I_n - A)^{-1}B + D)\rho$. In general $\tilde{O}_\infty$ is not finitely-determined, *i.e.*, it cannot be represented using a finite number of inequalities, and hence cannot be used to define a QP. Therefore we introduce an approximation to $\tilde{O}_\infty$ that can be used in its place. The approximation requires $V = \{\rho : y_{\mathrm{ss}}(\rho) \in Y\}$, which is the set of all steady-state admissible references. The approximation to $\tilde{O}_\infty$ is given by,

$$\tilde{P} = \{(x_0, \bar{x}_0, \rho) : (x_0, \bar{x}_0, \rho) \in \tilde{O}_\infty,\ \rho \in V'\}, \tag{5}$$

where $V' \in \operatorname{int} V$ is a polytopic approximation of V. The set $\tilde{P}$ can be computed in finite time [13] and, in general, the computation of $\tilde{P}$ becomes faster as V' becomes smaller.

In order to determine a constraint-admissible reference input, the ECG algorithm solves an optimization at each time-instant t. The variables to be optimized are $\bar{x}(t)$ and $\rho(t)$, and are determined by,

$$\begin{aligned}(\bar{x}(t), \rho(t)) = \underset{(\bar{x}, \rho)}{\arg\min}\ & \|\bar{x}\|_Q^2 + \|r(t) - \rho\|_R^2, \\ \text{sub. to } & (x(t), \bar{x}, \rho) \in \tilde{P}.\end{aligned} \tag{6}$$

The constraint-admissible reference input is set to the output of the auxiliary system (2),

$$v(t) = \bar{C}\bar{x}(t) + \rho(t).$$

The output of the ECG algorithm, *i.e.*, the solution to (6), exhibits three properties: **(a)** constraint-enforcement of the output constraint (1b), *i.e.*, $y(t) \in Y$ for all $t \in \mathbb{Z}_+$; **(b)** recursive feasibility of the solution to the optimization problem (6), *i.e.*, $(x(t), \bar{x}(t), \rho(t)) \in \tilde{P} \implies (x(t+1), \bar{x}(t), \rho(t)) \in \tilde{P}$; and **(c)** finite-time convergence to the closest steady-state constraint-admissible reference, *i.e.*, if for some finite $t_s \geq 0$, $r(t) = r$ for all $t \geq t_s$, then there exists a finite $t_c \geq t_s$ such that $v(t) = v$ for all $t \geq t_c$, where v minimizes $\|r - v\|_R^2$ subject to the constraint $v \in V'$.

III. DUAL-STAGE PROCESSING MACHINE

The problem considered in this paper is similar to that in [10]–[12]. A dual-stage positioning system is used to machine a sequence of desired target points onto a workpiece. The system is composed of a dual-axis slow stage and a dual-axis fast stage. The pattern to be machined on the workpiece is given by two sequences,

$$\{c_j^i\}_{j=1,\ldots,N}, \tag{7}$$

where $i \in \{x, y\}$, so that every (c_j^x, c_j^y) corresponds to a target point that must be machined. Figs. 1-2 show the pattern from three views: Fig. 1 shows the 2D view of the pattern $\{(c_j^x, c_j^y)\}$; Fig. 2 shows the sequences $\{c_j^x\}$ and $\{c_j^y\}$.

For both the x- and y-axes, the slow stage dynamics are the same. They are given by the linear dynamic equations,

$$\dot{x}_s^i(t) = A_s x_s^i(t) + B_s u_s^i(t), \tag{8a}$$

$$y_s^i(t) = C_s x_s^i(t) + D_s u_s^i(t). \tag{8b}$$

The system is controllable, observable, and asymptotically stable. The variables $x_s^i(t)$ are the states, $u_s^i(t)$ are the control inputs, and y_s^i are the positions. The system includes inner-feedback loops so that the position y_s^i approaches u_s^i whenever $u_s^i(t) \equiv u_s^i$ is held constant, *i.e.*, $\lim_{t\to\infty, u_s^i(t)=u_s^i} y_s^i(t) = u_s^i$. The controller sampling time is T_s.

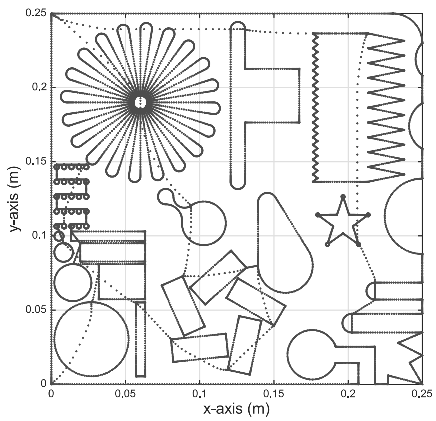

Fig. 1. Target pattern (7) (solid) and desired reference (19) (dotted)

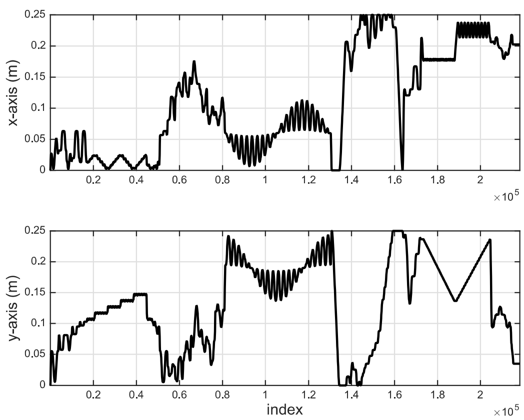

Fig. 2. Top: $\{c_j^x\}$; Bottom: $\{c_j^y\}$

The dynamics of the fast stage are also linear and are given by,

$$\dot{x}_f^i(t) = A_f x_f^i(t) + B_f(r^i(t) - \tilde{y}_s^i(t)), \tag{9a}$$

$$y_f^i(t) = C_f x_f^i(t) + D_f(r^i(t) - \tilde{y}_s^i(t)). \tag{9b}$$

The system is overdamped and designed so that the outputs $y_f^i(t)$ aggressively track $r^i(t) - \tilde{y}_s^i(t)$, where $\tilde{y}_s^i(t)$ are precise estimates of $y_s^i(t)$. The time constant of the fast dynamics is much faster than the time constant of the slow sampling time T_s. The variables $r^i(t)$ are reference inputs that precisely track the sequences of target points (7). Because the fast stage is fast and precise, $y_f^i(t)$ tracks $r^i(t) - y_s^i(t)$ almost instantaneously. Relative to the workpiece, the position of the fast stage is $y_s^i(t) + y_f^i(t)$. It is thus guaranteed, in the unconstrained case, that the overall system tracks $r^i(t)$ and the sequence (7) is machined as required.

A. Constraints

Both the slow and fast dynamics, (8) and (9) respectively, are constrained. The slow stage is limited in allowable magnitude of velocity and acceleration. The fast stage is designed to move quickly but within a limited range, so that the corresponding limitation is on its range of motion.

1) Slow Stage Constraints: Due to its size and weight, for safety and performance reasons, the slow stage must avoid aggressive motion. When the system is unconstrained, the tracking controller may allow for accelerations that are sometimes too large. For the slow stage, the constraints are on the position, velocity, and acceleration,

$$p_{\min} \le y_s^i(t) \le p_{\max}, \tag{10a}$$

$$v_{\min} \le \dot{y}_s^i(t) \le v_{\max}, \tag{10b}$$

$$a_{\min} \le \ddot{y}_s^i(t) \le a_{\max} \tag{10c}$$

for all t. In practice, the range of motion for the slow stage is large enough relative to the pattern (7) so that the position constraint rarely becomes active. The more stringent constraints are on the velocity and acceleration, (10b) and (10c) respectively.

2) Fast stage constraints: Because the fast stage is light, it can move aggressively. However, as a consequence its range of motion, its motion is limited to a small area. The constraint on the fast stage is given by,

$$|y_f^i(t)| \le \delta,$$

for all t, where δ is the stroke length, *i.e.*, the maximum allowable distance from the origin. Since the fast stage dynamics are over-damped, there is no overshoot; and since $y_f^i(t)$ closely tracks $r^i(t) - y_s^i(t)$, the above constraint is enforced whenever,

$$|r^i(t) - y_s^i(t)| \le \delta. \tag{11}$$

B. Slow stage tracking

The fast stage tracks the error $r^i(t) - y_s^i(t)$. As discussed in the introduction, it is not required that the slow stage track the target points (7). Since the error is limited by the constraint (11), all that is required is that the slow stage approach the desired pattern within a distance δ so that the fast stage is in position to machine it. For both the x- and y-axes, the position of the target points changes rapidly; attempting to track these points with the slow stage results in frequent short-distance movements and invariably results in slow processing. In the following section, we present a tracking scheme where the goal is to ensure that the slow stage always moves in the direction of future target points, while ensuring that the positions $y_s^i(t)$ are always within stroke length δ of the current target reference $r^i(t)$.

C. System design

A schematic of the system configuration is shown in Fig. 3. In it, a sequence of target points are generated and passed to a tracking algorithm, which generates a reference for the slow stage. This reference is modified by the ECG constraint enforcement scheme and passed to both the slow and fast stages to machine the desired pattern.

IV. Slow-stage tracking controller with guaranteed constraint enforcement

In this section, we describe the architecture of the control system that tracks the pattern (7), while achieving desired performance characteristics.

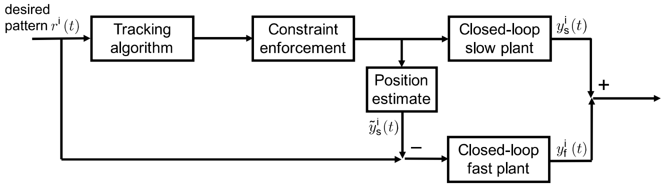

Fig. 3. System schematic

A. Constraint enforcement and tracking using ECGs

Both the slow and fast stages are subject to constraints (10) and (11), respectively, but must track a reference input in order to machine the pattern (7). Here we introduce an ECG approach that is designed to tightly enforce the slow-stage constraints (10), while ensuring that the constraint admissible reference $v_s^i(t_k)$ tracks the pattern (7) closely enough to satisfy the stroke constraint (11).

1) Slow-stage constraint enforcement: We begin by discussing the slow-stage constraints (10), which have been defined in continuous-time. The ECG is a scheme that enforces pointwise-in-time constraints, *i.e.*, at the sampling times $t_k, t_{k+1}, \ldots,$ and does not guarantee inter-sampling constraint enforcement. In order to minimize potential inter-sampling constraint violation, in our design we discretize the closed-loop system model (8) using the discretization time interval,

$$T_p = T_s/N,$$

which is a fraction $1/N$ of the slow-stage sampling time T_s. The resulting predictive model is of the form,

$$x_s^i(\tilde{t}_{n+1}) = A_p x_s^i(\tilde{t}_n) + B_p v_s^i(\tilde{t}_n), \tag{12a}$$
$$z_s^i(\tilde{t}_n) = C_p x_s^i(\tilde{t}_n) + D_p v_s^i(\tilde{t}_n) \in Y, \tag{12b}$$

where $\tilde{t}_{n+1} = \tilde{t}_n + T_p$. The constrained output is $z_s^i(\tilde{t}_n) = (y_s^i(\tilde{t}_n), \dot{y}_s^i(\tilde{t}_n), \ddot{y}_s^i)$, and $Y = [p_{\min}, p_{\max}] \times [v_{\min}, v_{\max}] \times [a_{\min}, a_{\max}]$ is the output constraint set.

The ECG is designed to take into account that $v_s^i(\tilde{t}_n)$ is constant over every N sampling periods from n to $n+\bar{n}N-1$. We define two shift register matrices [14]: an $\bar{n}N$-by-$\bar{n}N$ matrix $\bar{A}_p$ and a 1-by-$\bar{n}N$ matrix $\bar{C}_p$. The auxiliary reference dynamics are therefore,

$$\bar{x}^i(\tilde{t}_{n+1}) = \bar{A}_p \bar{x}^i(\tilde{t}_n), \tag{13a}$$
$$v_s^i(\tilde{t}_n) = \bar{C}_p \bar{x}^i(\tilde{t}_n) + \rho^i(\tilde{t}_n), \tag{13b}$$

The constraint set $\tilde{O}_\infty$ from (4) can now be defined,

$$\begin{aligned}&\tilde{O}_\infty = \{(x_0, \bar{x}, \rho) : x_s^i(\tilde{t}_0) = x_0,\\ &\quad \bar{x}^i_{kN-\ell}(\tilde{t}_0) = \bar{x}_k,\ k = 1, \ldots, \bar{n},\ \ell = 0, \ldots, N-1,\\ &\rho^i(\tilde{t}_n) = \rho,\ (12),\ (13) \text{ is satisfied for all } n \in \mathbb{Z}_+\}.\end{aligned} \tag{14}$$

Note that $\tilde{O}_\infty$ is the same for both the slow-stage x- and y-dynamics, because the dynamics and constraints are the same in both axes.

2) Fast-stage constraint enforcement: The stroke constraint (11) is enforced by the slow stage. It requires that the pattern (7) always stay within a stroke length δ of the worktool origin. Since multiple points c_j^i are machined between the slow-stage updates, we are required to enforce inter-sampling constraints. A method of doing this is to discretize the prediction model with a higher sampling rate, as was done in the case of the slow-stage. It is not practical to do so in this case, because the fast stage sampling rate T_f is much faster than the slow stage sampling rate, *i.e.*, the ratio between slow and fast time constants,

$$M = T_s/T_f \gg 1,$$

and choosing $N = M$ would result in a large burden on computing the constraint set $\tilde{O}_\infty$, since the number of inequalities needed to construct it would drastically increase.

An alternative is to follow an approach similar to [10]–[12] and make the realistic assumption that the position $y_s^i(t)$ stays bounded within sampling periods, *i.e.*,

$$\begin{aligned}\min(y_s^i(t_k), y_s^i(t_{k+1})) \le y_s^i(t) \le \max(y_s^i(t_k), y_s^i(t_{k+1})),\\ \forall t \in [t_k, t_{k+1}].\end{aligned} \tag{15}$$

Using this assumption, we design a method of choosing the points (7) in the pattern to be machined within a sampling period T_p, and constructing a set of constraints that ensures the adherence of the stroke constraint whenever the points are machined.

At every time t_k, given a predicted position response computed during the previous update, $y_s^i(t_{k|k-1}), y_s^i(t_{k+1|k-1}), \ldots, y_s^i(t_{k+\bar{n}+1|k-1})$, and given the index $j = j_{k-1}$ of the latest point in the sequence (7) that has been machined, the method is designed to attempt to maximize the number of points $\{c_j^i\}$ in the sequence that can be machined between t_k and $t_{k+\bar{n}+1}$. This is done by computing predicted indexes $j_{k+1|k}, \ldots, j_{k+\bar{n}+1|k}$, which are maximized based on the assumption (15).

3) ECG for tracking time-varying references: The input to the ECG scheme is a reference sequence of length $\bar{n}+1$, which represents the desired reference for the present sampling time, and the future $\bar{n}$ sampling periods. In the design of the ordinary ECG introduced previously, a main assumption is that the desired reference stays constant for all present and future time instants, but this does not need to be the case. To show this, assume that at time t_k, the desired sequence is,

$$r(t_k), r(t_{k+1|k}), \ldots, r(t_{k+\bar{n}|k}), \tag{16}$$

and let $\Delta r(t_k) = (r(t_k) - r(t_{k+\bar{n}|k}), \ldots, r(t_{k+\bar{n}-1|k}) - r(t_{k+\bar{n}|k}))$. Then the ECG optimization can be modified to,

$$\begin{aligned}&(\bar{x}^i(t_k), \rho^i(t_k)) =\\ &\qquad \arg\min_{(\bar{x},\rho)} \|\bar{x} - \Delta r(t_k)\|_Q^2 + \|r(t_{k+\bar{n}|k}) - \rho\|_R^2, \quad (17)\\ &\qquad \text{sub. to } (x_s^i(t_k), \bar{x}, \rho) \in \tilde{P},\\ &\qquad\qquad (y_s^i(t_{k|k}), \ldots, y_s^i(t_{k+\bar{n}|k})) \in S_k^i,\\ &\qquad\qquad (y_s^i(t_{k+1|k}), \ldots, y_s^i(t_{k+\bar{n}+1|k})) \in S_k^i,\end{aligned}$$

where the reference input is given by,

$$v_s^i(t_k) = \bar{C}\bar{x}^i(t_k) + \rho^i(t_k). \tag{18}$$

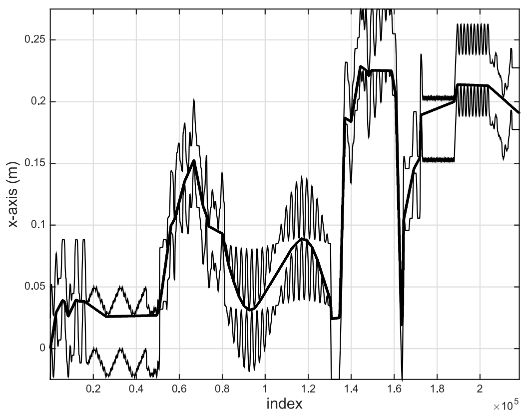

Fig. 4. Desired reference sequence $\{\hat{c}_j^x\}$

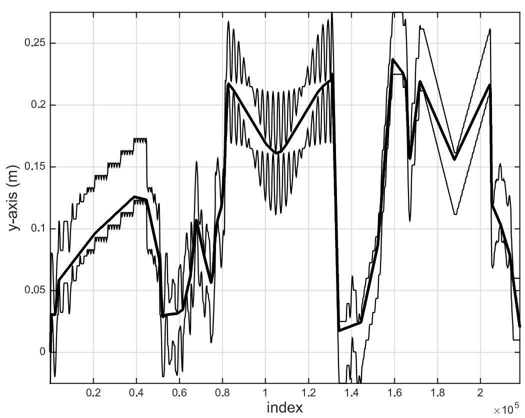

Fig. 5. Desired reference sequence $\{\hat{c}_j^y\}$

B. Trajectory generation for the slow stage

The final part of our design is the trajectory generation for the slow-stage axes. Here we present the construction of the sequence (16) to be tracked by the ECG. To obtain a reference trajectory, we modify the sequence of target points (7) to a sequence of desired references that tracks the target points and satisfies the stroke constraint (11).

The target sequence (7) varies very quickly and, due to the slow response properties of the slow stage, it is generally undesirable for the reference to track the sequence. As such, we wish to minimize the the distance covered by the slow stage. To do this, we generate two sequences of points,

$$\{\hat{c}_j^i\}_{j=1,\ldots,J}, \tag{19}$$

which minimize,

$$\sum_{j=1}^{J-1} |\hat{c}_{j+1}^i - \hat{c}_j^i|, \tag{20}$$

subject to the boundary conditions $\hat{c}_1^i = c_1^i$, $\hat{c}_J^i = c_J^i$, and the constraint $|\hat{c}_j - c_j| \leq \delta - \varepsilon$ for all j, where $\varepsilon > 0$ is a small parameter that has been introduced in order to guarantee that the reference does not coincide with the hard constraint and convergence to any $\hat{c}_j$ is always possible.

Solutions to (20) minimize the distance covered by the sequence (19), and are not unique in general. From the possibilities, we use a sequence that stays away from the envelope boundaries as much as possible, and which follows a straight-line path when it does not coincide with a constraint boundary. The sequences corresponding to this approach are plotted in Figs. 4-5. The 2D view of the reference is given as the dotted line in Fig. 1. The desired reference is then obtained using the predicted indexes $j_{k+\ell|k}$,

$$r^i(t_{k+\ell}|t_k) = \hat{c}^i_{\min(j(k+\ell|k)+1,J)}, \tag{21}$$

so that the desired reference matches the first point outside of the sequence to be machined within the current sampling period. This ensures that the slow stage always tends towards future points in the target sequence and away from the points that will be machined.

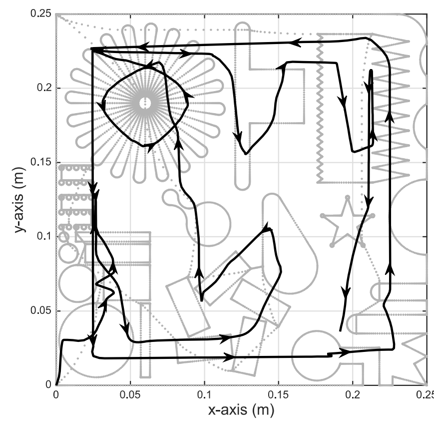

Fig. 6. Response of $(y_s^x(t), y_s^y(t))$ plotted against pattern (7)

V. Numerical simulation

We perform a numerical simulation for a two-stage machine with sampling time $T_s = 35.6$ms and time-scale separation corresponding to $M = 500$. The slow-stage controller is designed with a prediction horizon of $\bar{n} = 40$ and $N = 3$ is chosen in order to reduce the sampling rate. The ECG penalty matrices are given by $\bar{Q} = I_{\bar{n}N}$ and $\bar{R} = 0.01$. The constraints (10) are given by $p_{\min} = 0$ m, $p_{\max} = 0.25$ m, $v_{\min,\max} = \pm 0.1$ m/s, and $a_{\min,\max} = \pm 10$ m/s^2. The stroke constraint (11) is given by $\delta = 0.025$. The results are reported in Figs. 6-10.

Fig. 6 shows the response of the slow-stage position relative to the pattern. Figs. 7-8 show the response of the positions in the x- and y-axes plotted against the associated stroke constraints. Comparing these to Figs. 4-5, we can see that the two match each other closely. Fig. 9 shows the response of the slow-stage velocities and accelerations, for which the constraints are enforced. Fig. 10 shows the number of machined points during each time sample; note that no more than M points can be machined during the sampling period. The pattern is machined in a total of 30.12 seconds, during which time, as the results show, the ECG scheme enforces all system constraints.

VI. Conclusion

In this paper, we presented a constraint-enforcement scheme for precision manufacturing. The system consists of

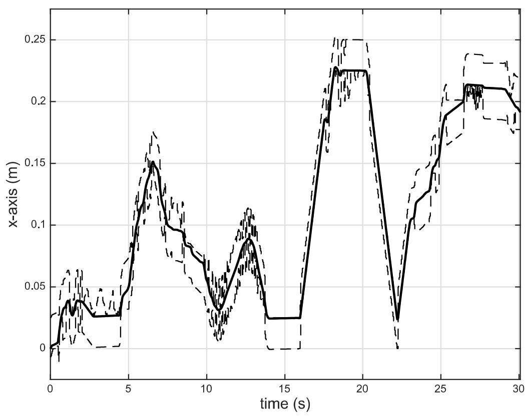

Fig. 7. Response of $y_s^x(t)$ (solid) plotted against $c_{\max}^x(t)$ and $c_{\min}^x(t)$ (dotted)

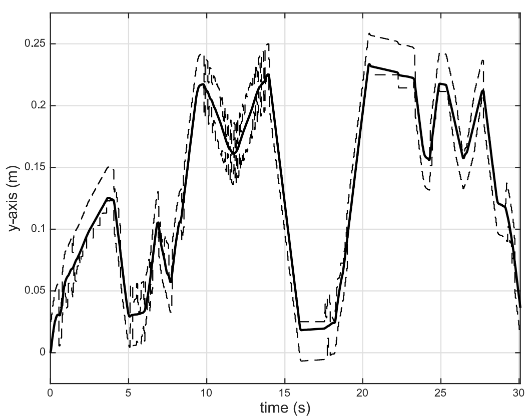

Fig. 8. Response of $y_s^y(t)$ (solid) plotted against $c_{\max}^y(t)$ and $c_{\min}^y(t)$ (dashed)

two stages with large time-scale separation. The slow stage controller enforces both slow- and fast-stage constraints so that a desired pattern may be machined by the fast-stage. The scheme that we have presented is based on an extended command governor that has been modified to ensure close tracking of the pattern. Numerical results were reported, showing successful tracking and constraint enforcement.

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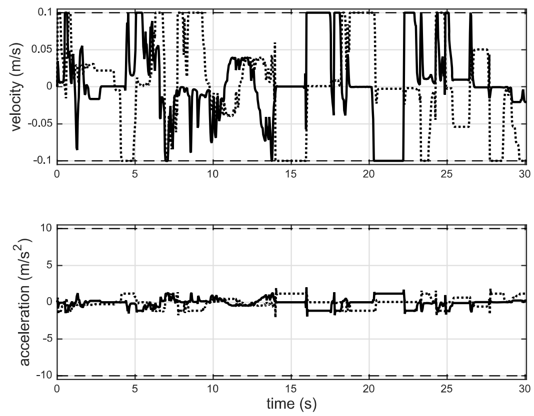

Fig. 9. Responses of $\dot{y}_s^x(t)$ (top, solid), $\dot{y}_s^y(t)$ (top, dotted), $\ddot{y}_s^x(t)$ (bottom, solid), and $\ddot{y}_s^y(t)$ (bottom, dotted) plotted against constraints (dashed)

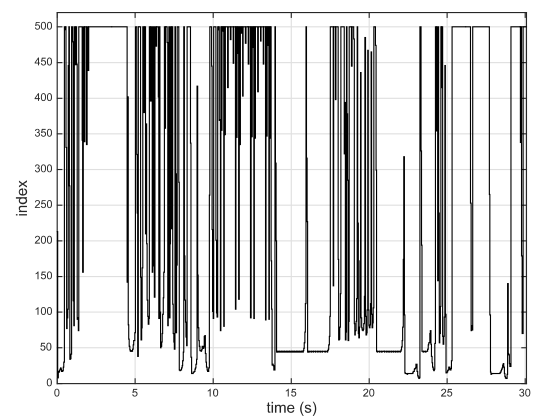

Fig. 10. Response of $j(t)$

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